Supplementary information - Temporal integration by calcium dynamics in a model neuron

The supplementary information provides details on the spiking neuron model used in this work.

The model is based on the single-compartment spiking neuron model[1] with the addition of a non-specific calcium dependent cationic current[2]. The currents equation is:

$$C\frac{dV}{dt} = -I_L - I_{Na} - I_K - I_A - I_{cat}$$

The leak current is given by $I_L = g_L(V - E_L)$. The sodium I_{Na} and delayed rectifier I_K currents are described in a standard way: $I_{Na} = \bar{g}_{Na} m_\infty^3 h(V - E_{Na})$ for the sodium current and $I_K = \bar{g}_K n^4 (V - E_K)$ for the delayed rectifier current. The gating variables $x = h, n$ satisfy the relaxation equations: $dx/dt = (x_\infty - x)/\tau_x$. The functions x_∞, and τ_x are: $x_\infty = \alpha_x/(\alpha_x + \beta_x)$, and $\tau_\infty = \phi/(\alpha_x + \beta_x)$ where $\phi = 0.1$; α_x, β_x in units of 1/sec are: $\alpha_h = 70\exp(-(V+44)/20)$, $\beta_h = 1000/(\exp(-0.1(V+14))+1)$, $\alpha_n = -10(V+34)/(\exp(-0.1(V+34))-1)$ and $\beta_n = 125\exp(-(V+44)/80)$. The activation term $m_\infty = \alpha_m/(\alpha_m + \beta_m)$ where $\alpha_m = -100(V+30)/(\exp(-0.1(V+30))-1)$, $\beta_m = 4000\exp(-(V+55)/18)$. V is in mV.

The A-current is: $I_A = \bar{g}_A a_\infty^3 b(V - E_K)$ with $a_\infty = 1/(\exp(-(V+50)/20)+1)$. The inactivation term b satisfies $db/dt = (b_\infty - b)/\tau_A$ with $b_\infty = 1/(\exp((V+80)/6)+1)$ and $\tau_A = 0.02$ s is voltage independent. The other parameters of the model are: $C = 1$ μF/cm^2, $\bar{g}_{Na} = 10^5$ μS/cm^2, $\bar{g}_K = 4\cdot 10^4$ μS/cm^2, $g_L = 50$ μS/cm^2,

$\bar{g}_A = 2\cdot 10^4\ \mu\text{S/cm}^2$. The reversal potentials of the ionic currents are: $E_{Na} = 55\ \text{mV}$, $E_K = -80\ \text{mV}$, and $E_L = -65\ \text{mV}$.

The dependence of the membrane potential dynamics on $[Ca^{+2}]_i$ results from the non-specific calcium dependent cationic current $I_{cat} = g_{cat}(V - E_{cat})$, with $E_{cat} = 0\ \text{mV}$. We assume that non-specific calcium dependent cationic channels are homogeneously distributed along the dendrite, and that their conductance density is dependent on the local $[Ca^{+2}]_i$. The mean conductance is:

$$g_{cat} = \bar{g}_{cat} \frac{\int d\vec{r} F[c(\vec{r})]}{\int d\vec{r}}$$

where $c(\vec{r})$ is the local $[Ca^{+2}]_i$, $\bar{g}_{cat}$ is the maximal conductance, $F(c)$ is a calcium-dependent activation term and the integral is over the membrane area of the dendrite. Here $F[c(\vec{r})] = c(\vec{r}) - c_1$ and $\bar{g}_{cat} = 300\ \mu\text{S/cm}^2/\mu\text{M}$. Assuming a long dendrite, the shape of the front is independent of its location and therefore I_{cat} is proportional to the location of the front along the dendrite.

The firing rate of the Shriki model[1] for spiking neurons is nearly proportional to the total conductance. Therefore, the firing rate of our model cell is approximately proportional to the location of the calcium front.

Note that this model assumes that the neuron is electrically compact but calcium distributed. This results from the large difference in the diffusion coefficient between calcium and membrane potential. The effective calcium concentration diffusion coefficient in neurons is $D_{Ca} \approx 10 - 10^2\ \mu\text{m}^2/\text{s}$ (ref. 3). In contrast, the membrane potential diffusion coefficient is much larger, on the order of $D_V \approx 10^8\ \mu\text{m}^2/\text{s}$ (ref 4). The characteristic time scales of membrane potential and calcium dynamics are

$\tau_V \approx 10^{-3} - 10^{-2}$ s and $\tau_{Ca} \approx 10^{-1} - 10^{0}$ s, respectively. Hence, the resulting characteristic electrotonic length of dendrites is $\lambda_V \approx 10^{2} - 10^{3}$ μm, much larger than the characteristic "calcium length", which is $\lambda_{Ca} \approx 10^{0} - 10^{1}$ μm. Here we assume that the length of the dendrites is shorter than the electrotonic length, but longer than the calcium length, making the neuron electrically compact but calcium distributed.

References:

1. Shriki, O., Hansel, D. & Sompolinsky, H. *Neural Computation* in press (2003).

2. Egorov, A. V., Hamam, B. N., Fransen, E., Hasselmo, M. E. & Alonso, A. A. Graded persistent activity in entorhinal cortex neurons. *Nature* **420**, 173-8. (2002).

3. Allbritton, N. L., Meyer, T. & Lubert, S. Range of messenger action on calcium ion and inositol 1, 4, 5-trisphosphate. *Science* **258**, 1812-1815 (1992).

4. Tuckwell, H. C. *Introduction to theoretical neurobiology* (Cambridge University Press, Cambridge, 1988).