Competitive interactions in nonlinear networks

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Multistability and hysteresis are seen in perception.

1 Mutual inhibition between two neurons

In our discussion of feedback in linear networks, we covered the case of two neurons interacting by mutual inhibition. This example showed that inhibition could mediate positive feedback. In our discussion of Limulus retina, we added rectification nonlinearity. Let us return to this example,

\[ \begin{align*}
\dot{x}_1 + x_1 &= [b_1 - \beta x_2]^+ \\
\dot{x}_2 + x_2 &= [b_2 - \beta x_1]^+ 
\end{align*} \]

Suppose that \( \beta < 1 \). Then there are three nonoverlapping regions of input space, in each of which there is a unique steady state.

1. If \( b_1/b_2 < \beta \), then there is a steady state with only neuron two active,

\[ x_1 = 0 \quad x_2 = b_2 \]

2. If \( \beta < b_1/b_2 < \beta^{-1} \), then both neurons are active

\[ x_1 = \frac{b_1 - \beta b_2}{1 - \beta^2} \quad x_2 = \frac{b_2 - \beta b_1}{1 - \beta^2} \]

3. If \( \beta^{-1} < b_1/b_2 \), then there is a steady state with only neuron one active,

\[ x_1 = b_1 \quad x_2 = 0 \]

Suppose that \( \beta > 1 \)

1. If \( b_1/b_2 < \beta \), then there is a steady state with only neuron two active,

\[ x_1 = 0 \quad x_2 = b_2 \]
2. If $\beta^{-1} < b_1 / b_2 < \beta$, there is a steady state with both neurons active

\[
x_1 = \frac{b_2 - b_1}{\beta^2 - 1}, \quad x_2 = \frac{\beta b_1 - b_2}{\beta^2 - 1}
\]

3. If $\beta^{-1} < b_1 / b_2$, then there is a steady state with only neuron one active,

\[
x_1 = b_1, \quad x_2 = 0
\]

In this case, if $b_1 / b_2 < \beta^{-1}$, then there is a single steady state with only neuron two active. If $\beta^{-1} < b_1 / b_2 < \beta$, all three steady states exist. If $b_1 / b_2 > \beta$, then there is a single steady state with only neuron one active.

2 Self-excitation and global inhibition

\[
\begin{align*}
\dot{x}_i + x_i &= \left[ b_i + \alpha x_i - \beta y \right]^+ \\
\dot{y} + y &= \sum_j x_j
\end{align*}
\]

If $\alpha = \beta$ and $N = 2$, this reduces to the previous case

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rough correspondence between these ideas, but not the same inhibition and excitation
(anatomy) negative and positive feedback (linear systems theory) competition and co-operation

4 Stereopsis