Early vision: linear filters

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1 Receptive field

- Hubel-Wiesel video illustrates antagonistic center-surround responses and orientation selectivity.
- Definition: area in visual field where a spot of light (or dark) can evoke an elevated or suppressed neural response.
- Visual neurons have been interpreted in two ways: feature detectors and linear filters.
- Receptive fields become larger at later stages in the visual system. Also, selectivity becomes more complex.

2 Retinal ganglion cells

- send their axons out of the retina through the optic nerve.
- uniform illumination, or even complete darkness: background firing rate of 1-20 Hz
- on-center has maximal response to white spot on black background.
- off-center has maximal response to black spot on white background.
- antagonistic surround
- with a spot covering the entire receptive field, the center just barely wins, or the center and surround cancel each other out.

3 Difference of Gaussians (DOG) model

- For the case where the center and surround cancel each other, a simple model for the receptive field is

$$K(x) = \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left(-\frac{x^2}{2\sigma_c^2}\right) - \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{x^2}{2\sigma_s^2}\right)$$

- In this idealized case, both the center and surround are normalized to unit area. In reality, the parameters would be fit to experimental measurements.
- alternative: Laplacian of a Gaussian

4 Fourier analysis (continuous)

- The response of a population of ganglion cells can be modeled as a convolution:

$$O(x) = \int dx' K(x - x') I(x')$$

Let's work in one dimension for simplicity.

- Define the Fourier transform

$$\tilde{O}(q) = \int dx \, e^{iqx} O(x)$$

so that

$$\tilde{O}(q) = \tilde{K}(q) \tilde{I}(q)$$

- Fourier transform of DOG

$$\tilde{K}(q) = \exp\left(-\frac{\sigma_c^2 q^2}{2}\right) - \exp\left(-\frac{\sigma_s^2 q^2}{2}\right)$$

- This vanishes at $q = 0$ and as $q \to \infty$, so it's a band-pass filter. Eliminating low spatial frequencies means that only differences in illumination cause a response, not the overall level. However, the high spatial frequencies are also eliminated, so that noise is suppressed.

5 Fourier analysis (discrete)

- See Press et al., *Numerical Recipes* and `www.nr.com`
- $O_i = \sum_j K_{ij} I_j$
- With periodic boundary conditions, a discrete convolution means that K_{ij} is a circulant matrix, meaning that the entries depend only on $i - j$. In the homework, you will show that the eigenvectors of K are Fourier modes.
- The Fourier decomposition is actually a special case of matrix diagonalization.

6 Eigenmode analysis (matrix diagonalization)

- Suppose that $O_i = \sum_j K_{ij} I_j$, but K is not a circulant matrix, but it is square.
- Diagonalize K as $S \Lambda S^{-1}$, where Λ is a diagonal matrix. This implies that $S^{-1}y = \Lambda S^{-1}x$. So the columns of S (eigenmodes) are a basis, and multiplying by S^{-1} yields the coordinates in this basis.
- The eigenvalues are the gains of amplification or attenuation for each eigenmode.
- Special case: $K = K^T$. Then $S^{-1} = S^T$ (orthogonal matrix), and the eigenvalues are real.

7 Singular value decomposition

- If K is not square, write it as $K = Q_1 \Lambda Q_2^T$, where Q_1 and Q_2 are orthogonal matrices, and Λ is a diagonal matrix.
- The columns of Q_1 and Q_2 are bases in the input and output spaces.
- The singular values (diagonal elements of Λ) quantify the amount of coupling between the basis vectors.

8 Edge detection

- What is the computational function of the center-surround receptive field? According to the filtering idea, it acts as a band-pass filter. Researchers in computer vision have proposed another explanation, that center-surround processing is useful in edge detection. David Marr, among others, argued that edge detection would be a useful first step in any vision algorithm. This was part of his "primal sketch" idea.
- Edge detectors are generally based on some kind of derivative operation.
  - If an edge is a step in intensity,
  - then it should be marked by a peak in the first derivative,
  - or a zero-crossing in the second derivative.
- The second derivative idea is used in the Marr-Hildreth algorithm
  - This is based on filtering with a Laplacian of a Gaussian

    $$G(x) = \exp(-|x|^2/(2\sigma^2)) \qquad K = \nabla^2 G$$

  - convolve with K

    $$O = K * I = \int dx' K(x - x') I(x')$$

    and look for zero crossings

- combination of two operations
  - differentiation (to find edges)
  - blurring (to minimize effect of noise)
  - $(\nabla^2 G) * I = \nabla^2 (G * I) = G * (\nabla^2 I)$
- show examples
- Fourier domain
  - convolution becomes multiplication

$$\tilde{O}(k) = \tilde{K}(k)\tilde{I}(k)$$

  - each Fourier component of the input is multiplied by some factor
  - The Fourier transform of G is another Gaussian.
  - The Fourier transform of ∇^2 is $-k^2$.
  - put them together, get band pass filter
    * low frequencies out (differentiation)
    * high frequencies out (blurring)
- biological implementation
  - $\nabla^2 G$ is center-surround receptive field retinal ganglion cells LGN cells
  - AND of on-center and off-center cells gives zero-crossing detector